Appendix F
Accelerometer Signal Processing and ISO Index Computations

Introduction

ISO International Standard [3], Part 1: "General Requirements" and Part 5: "Method for evaluation of vibration containing multiple shocks," contains procedures for computing indices which can be used to evaluate the effect of human whole-body vibration on human health, comfort, and safety. Part 1 of the standard specifies three indices applicable to vibration in six degrees of freedom acting on, with applicable weighting factors, standing, seated, and recumbent bodies. Vertical vibration acting on a seated person is the only case of interest in the current work and this description of the application of the ISO procedures is for that case only. Part 5 of ISO 2631 provides a single index for vertical acceleration acting on a seated person. The four indices are as follows, with the first three from Part 1 and the fifth from Part 5.

1. Basic evaluation method using weighted root-mean-square (RMS) acceleration.
2. Running RMS method.
3. Fourth power vibration dose method.
4. Spinal response acceleration dose method.

A frequency weighting (filter) function must be applied to the raw acceleration signal before calculation of the first three indices. The fourth index does not require the application of a frequency weighting function. In addition, a weighted acceleration signal crest factor is defined for determining when the second through fourth indices might be applicable as a supplement to the basic RMS method. The applicable section of ISO 2631 is reproduced below in sufficient detail to define the procedure required to calculate each of the indices, followed by a detailed description of the procedure developed to weight the acceleration records measured in the cockpit of the simulator.

Index Definitions

1. Weighted RMS, reproduced from ISO 2631 Part 1, units = m/s^2

$$a_{\mathrm{W}} = \left[\frac{1}{T}\int_0^T a_{\mathrm{W}}^2(t)\,\mathrm{d}t\right]^{\frac{1}{2}}$$

Where

$a_{\mathrm{W}}(t)$ is the weighted acceleration (translational or rotational) as a function of time (time history), in meters per second squared (m/s^2) or radians per second squared (rad/s^2), respectively;

T is the duration of the measurement, in seconds.

2. From running RMS , reproduced from ISO 2631 Part 1, units = m/s^2

$$a_{\mathrm{W}}(t_0) = \left\{ \frac{1}{\tau} \int_{t_0-\tau}^{t_0} [a_{\mathrm{W}}(t)]^2 \, \mathrm{d}t \right\}^{\frac{1}{2}}$$

the maximum transient vibration value, MTVV, is defined as

MTVV = max[$a_{\mathrm{W}}(t_0)$]

i.e. the highest magnitude of $a_{\mathrm{W}}(t_0)$ read during the measurement period (T).

3. Fourth Power Vibration Dose (reproduced from ISO 2631 Part 1, units = m/s$^{1.75}$) is

$$\mathrm{VDV} = \left\{ \int_0^T [a_{\mathrm{W}}(t)]^4 \, \mathrm{d}t \right\}^{\frac{1}{4}}$$

Where

$a_{\mathrm{W}}(t)$ is the weighted acceleration (translational or rotational) as a function of time (time history), in meters per second squared (m/s^2) or radians per second squared (rad/s^2), respectively;

T is the duration of the measurement, in seconds.

4. Spinal Response Acceleration Dose (reproduced from ISO 2631 Part 5, units = m/s^2)

5.2.3 Spinal response in vertical direction (z-axis)

In the z-direction, the spinal response is non-linear and is represented by a recurrent neural network model.

The basis for this modeling technique is discussed in Annex C. Lumbar spine z-axis acceleration , a_{lz}, in meters per second squared is predicted using the following equations:

$$a_{lz}(t) = \sum_{j=1}^{7} W_j u_j(t) + W_8 \tag{2}$$

$$u_j(t) = \tanh\left[\sum_{i=1}^{4} w_{ji} a_{lz}(t-i) + \sum_{i=5}^{12} w_{ji} a_{sz}(t-i+4) + w_{j13} \right] \tag{3}$$

The model coefficients in Equations (2) and (3) are specific to a sampling rate of 160 per second. Therefore, data collected at a different sampling rate shall be resampled to 160 samples per second.

5.3 Calculation of acceleration dose

The acceleration dose, D_k, in meters per second squared, in the k-direction is defined as

$$D_k = \left[\sum_i A_{ik}^6\right]^{1/6}$$

Where

A_{ik} is the ith peak of the response acceleration $a_{lk}(t)$;
$k = x, y,$ or z.

A MATLAB computer program is provided in the standard for solving equations (2) and (3) above. The MATLAB program was translated into Visual Basic for computation of the acceleration dose. A sample time history is also provided in the standard with the corresponding spinal response dose output results for checking independent implementations of the model. See the description of the Visual Basic implementation given below.

Frequency Weighting

The filter for weighting the acceleration signals is implemented as a set of four differential equations defined by their frequency response functions. The frequency response functions are defined in the standard as four separate sections and these have to be transformed into the base differential equations for solution in the time domain. The response functions, as defined in the standard, and the base differential equations are shown below with the following nomenclature.

p = Laplace operator
$H(p)$ = Laplace transform
$|H(p)|$ = frequency response function
ω = frequency in rad/s
f = frequency in Hz
= $\omega / 2\pi$
ω_i, f_i, Q_i = response shaping parameters
x = input to the filter, acceleration in m/s^2
y = weighted output from the filter, acceleration in m/s^2

1. Band limiting - high pass section

Frequency response function:

$$|H_h(p)| = \left|\frac{1}{1 + \sqrt{2}\,\omega_1/p + (\omega_1/p)^2}\right| = \sqrt{\frac{f^4}{f^4 + f_1^4}}$$

Differential equations for numerical solution in terms of y and z:

$$\begin{aligned}
\ddot{y} &= \ddot{x} - \sqrt{2}\omega_1\dot{y} - \omega_1^2 y \\
\dot{y} &= z \\
\dot{z} &= \ddot{y} = \ddot{x} - \omega_1\left(\sqrt{2}z + \omega_1 y\right)
\end{aligned}$$

2. Band limiting - low pass section

Frequency response function:

$$|H_{\mathrm{l}}(p)| = \left|\frac{1}{1 + \sqrt{2}\,p/\omega_2 + (p/\omega_2)^2}\right| = \sqrt{\frac{f_2^4}{f^4 + f_2^4}}$$

Differential equations for numerical solution in terms of y and z:

$$\begin{aligned}
\ddot{y} &= \omega_2^2 x - \sqrt{2}\omega_2\dot{y} - \omega_2^2 y \\
\dot{y} &= z \\
\dot{z} &= \ddot{y} = \omega_2\left(\omega_2 x - \sqrt{2}z - \omega_2 y\right)
\end{aligned}$$

3. Acceleration-velocity transition (proportionality to acceleration at lower frequencies and proportionality to velocity at higher frequencies)

Frequency response function:

$$|H_{\mathrm{t}}(p)| = \left|\frac{1 + p/\omega_3}{1 + p/(Q_4\omega_4) + (p/\omega_4)^2}\right| = \sqrt{\frac{f^2 + f_3^2}{f_3^2}} \cdot \sqrt{\frac{f_4^4 \cdot Q_4^2}{f^4 \cdot Q_4^2 + f^2 \cdot f_4^2(1 - 2Q_4^2) + f_4^4 \cdot Q_4^2}}$$

Differential equations for numerical solution in terms of y and z:

$$\begin{aligned}
\ddot{y} &= \frac{\omega_4^2}{\omega_3}\dot{x} - \omega_4^2 x - \frac{\omega_4}{Q_4}\dot{y} - \omega_4^2 y \\
\dot{y} &= z \\
\dot{z} &= \ddot{y} = \omega_4\left(\frac{\omega_4}{\omega_3}\dot{x} + \omega_4 x - \frac{1}{Q_4}z - \omega_4 y\right)
\end{aligned}$$

4. Upward step (steepness approximately 6 dB per octave, proportionality to jerk)

Frequency response function:

$$|H_{\mathrm{s}}(p)| = \left|\frac{1 + p/Q_5\omega_5 + (p/\omega_5)^2}{1 + p/(Q_6\omega_6) + (p/\omega_6)^2} \cdot \left(\frac{\omega_5}{\omega_6}\right)^2\right| = \frac{Q_6}{Q_5} \cdot \sqrt{\frac{f^4 \cdot Q_5^2 + f^2 \cdot f_5^2(1 - 2Q_5^2) + f_5^4 \cdot Q_5^2}{f^4 \cdot Q_6^2 + f^2 \cdot f_6^2(1 - 2Q_6^2) + f_6^4 \cdot Q_6^2}}$$

Differential equations for numerical solution in terms of y and z:

$$\ddot{y} = \ddot{x} + \frac{\omega_5}{Q_5}\dot{x} + \omega_5^2 x - \frac{\omega_6}{Q_6}\dot{y} - \omega_6^2 y$$
$$\dot{y} = z$$
$$\dot{z} = \ddot{y} = \ddot{x} + \frac{\omega_5}{Q_5}\dot{x} + \omega_5^2 x - \frac{\omega_6}{Q_6} z - \omega_6^2 y$$

All of the frequency response functions except one require differentiation of the input signal. The following difference equations are used to differentiate numerically:

$$\dot{x} \approx \frac{\Delta x}{\Delta t} = \frac{x(t+h) - x(t-h)}{2h}$$
$$\ddot{x} \approx \frac{\Delta^2 x}{\Delta t^2} = \frac{x(t+h) - 2x(t) + x(t-h)}{h^2}$$

Where h = sample spacing = 1 / (sample rate)

Computer Implementation of the Weighting Functions

Each of the four weighting functions is implemented in a separate subroutine in a Visual Basic computer program. The subroutines are called in series with the measured cockpit acceleration used as input to the first subroutine and the output from the first subroutine fed into the input of the second subroutine, and so on through all four subroutines. The subroutines are called in the following order:

1. Low pass section.
2. Acceleration-velocity transition.
3. High pass section.
4. Upward step.

The order was selected primarily to reduce inaccuracies at high frequency during differentiation of the input. The low pass section does not require differentiation and attenuates the high frequencies before differentiation in the following sections. The acceleration-velocity transition filter requires only first order differentiation so that section follows the low pass. The other two filters require second order differentiation so they are executed last.

The overall transfer function is calculated from the frequency response functions as:

$$|H_{\text{Overall}}(p)| = |H_l(p)| \,.\, |H_t(p)| \,.\, |H_h(p)| \,.\, |H_s(p)|$$

The frequency response shaping parameters are specified in the ISO standard as shown in the following table:

Table A.1 — Parameters of the transfer functions of the principal frequency weightings

Weighting	Band-limiting		Acceleration-velocity transition (a-v transition)			Upward step			
	f_1 Hz	f_2 Hz	f_3 Hz	f_4 Hz	Q_4	f_5 Hz	Q_5	f_6 Hz	Q_6
W_k	0,4	100	12,5	12,5	0,63	2,37	0,91	3,35	0,91
W_d	0,4	100	2,0	2,0	0,63	∞	—	∞	—
W_f	0,08	0,63	∞	0,25	0,86	0,062 5	0,80	0,1	0,80

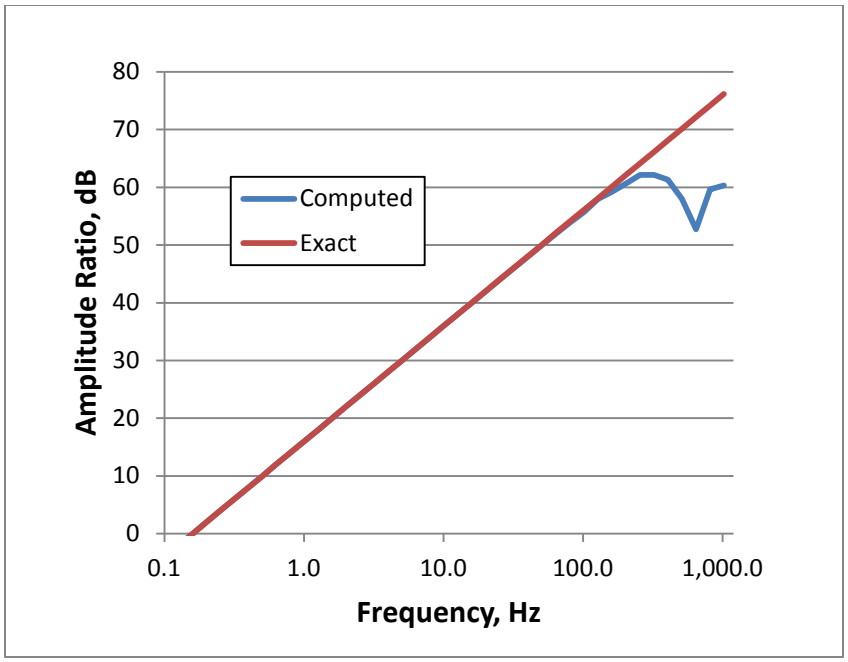
Weighting W_k is applicable to the present study and is for vertical motion of a seated subject.

Also, $\omega_i = 2\pi.f_i$

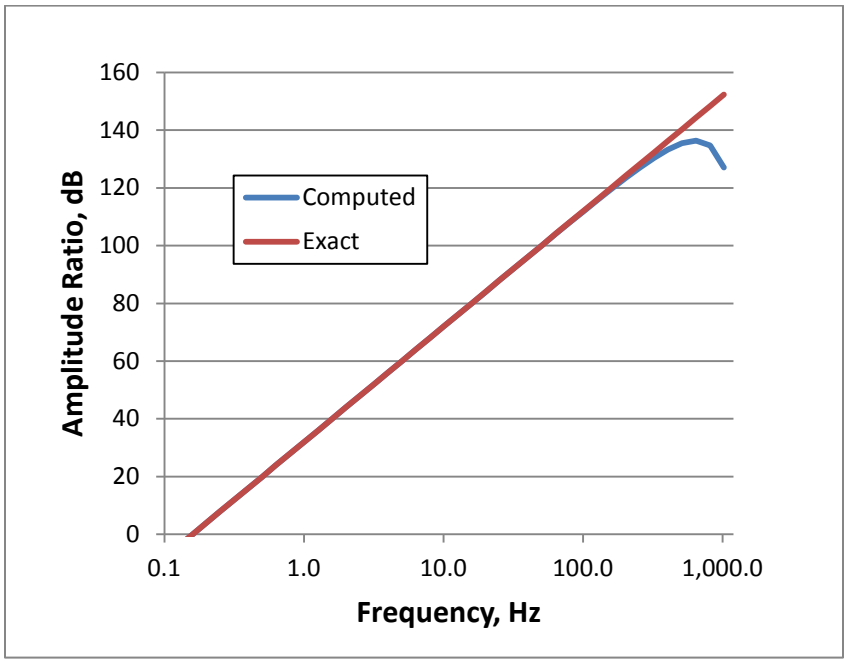
The principal steps in the weighting procedure are:

1. Read the accelerometer signal from an external data file. The 737-800 simulator sample rate is 60 Hz.
2. Fit cubic splines through the accelerometer data points and interpolate to a sample rate of 1,280 Hz.
3. Smooth the first one-second of the data record to suppress start-up transients during filtering.
4. Apply the weighting functions in series as explained above.
5. Fit cubic splines through the weighted data record and decimate down to a sample rate of 160 Hz.

The weighting function equations are solved using Runge-Kutta integration. The sample rate is increased from 60 Hz to 1,280 Hz to minimize numerically induced distortions over the frequency range of interest (about 0.02 to 100 Hz). The final sample rate of 160 Hz was selected because the Spinal Response Acceleration Dose index must be computed at a sample rate of 160 Hz and the same rate was used to compute the weighted indices for compatibility. Initially increasing the sample rate is particularly important in differentiating the input signals, as illustrated in the following figures.

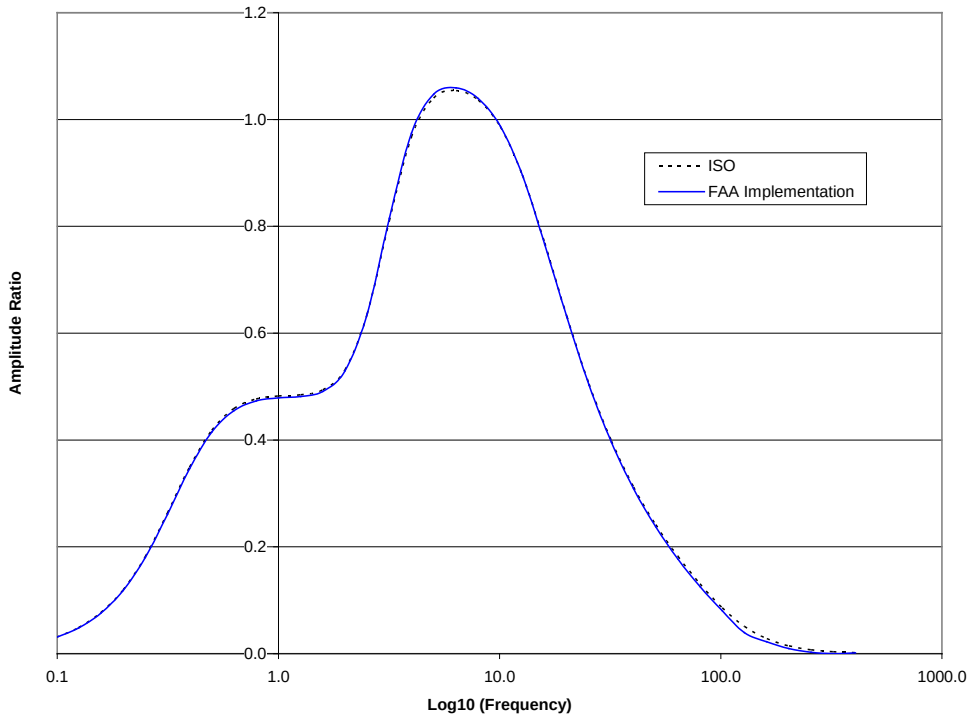


Computed and exact amplitude ratios for first order differentiation.

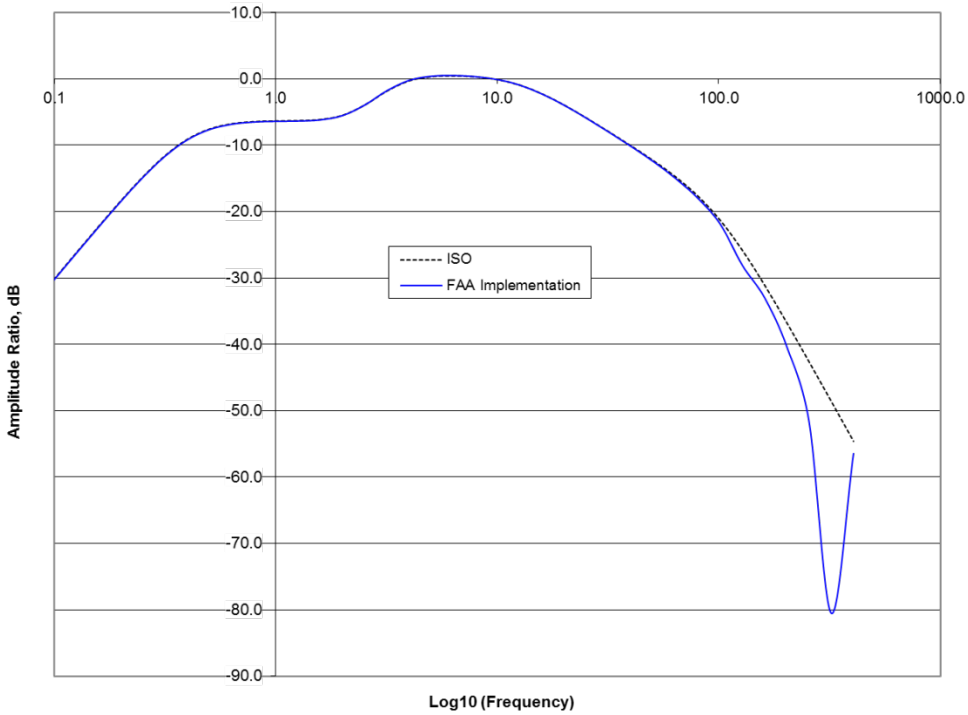


Computed and exact amplitude ratios for second order differentiation.

The frequency response of the weighting procedure is shown in the figure below. The curve marked “ISO” was produced by direct computation from the frequency transfer functions given above. The curve marked “FAA implementation” was produced by running unit amplitude sine waves through the weighting function computer program and plotting the amplitudes of the output sine waves.

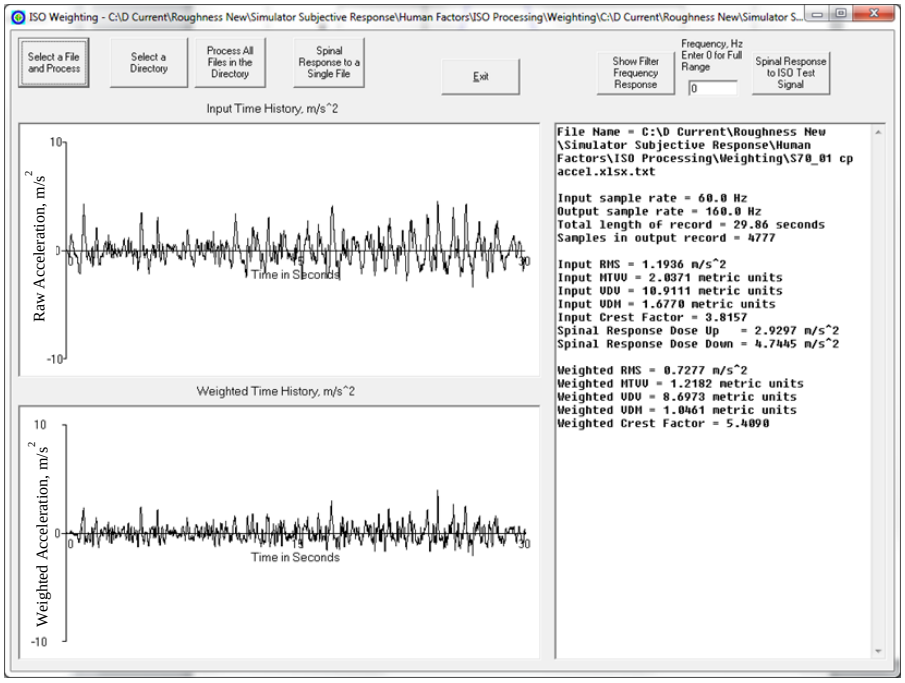


Frequency response of the weighting procedure compared with the ISO specified frequency transfer function. Amplitude ratio plotted on a linear scale.

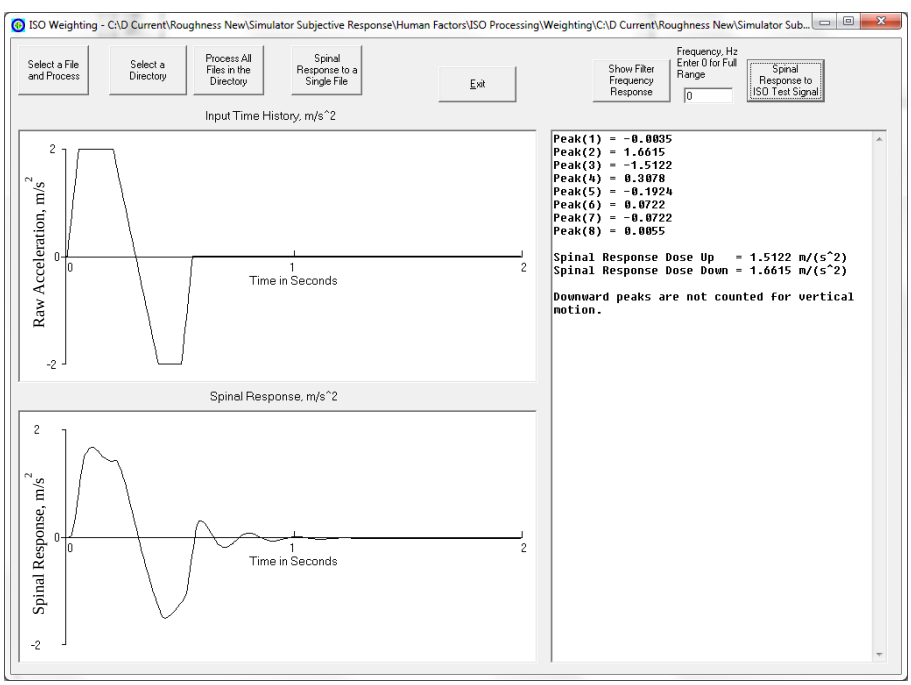


Frequency response of the weighting procedure compared with the ISO specified frequency transfer function. Amplitude ratio plotted on a logarithmic scale.

The next figure is a screen shot of the VB computer program showing a raw acceleration data record and the final weighted data record. Computed index values are also shown in the right pane.



The final figure is a screen shot showing the input and output for the check example of the Spinal Response Acceleration Dose procedure given in the ISO standard. The output shown in the lower left pane was calculated with the VB implementation of the ISO MATLAB program. The peak values and the dose values are shown in the right pane.



A copy of the source code of the computer program for weighting acceleration signals and computing the index values is available on request from the FAA Airport Technology R&D Branch, ANG-E260. A version of the program implemented in Microsoft Excel is also available.