**Appendix H**
**MATLAB Code to Generate Shifted Logarithmic Fits**

```
% Program logshift computes a shifted logarithmic fit y = a ln(x + s) + b of
% y = pilot rating of runway vs. x = ISO roughness parameters:
%   i.  Weighted RMS
%   ii. Weighted VDV, iii. DKup, iv. MTVV

% Display header:
fprintf(1,'\n\n----------------\n');
fprintf(1,'Program logshift\n');
fprintf(1,'----------------\n\n');
plot_type=input('Plot avg. ratings & conf. (1), indiv. ratings (2), or avg. ratings w/o conf. (3)?','s');
if size(plot_type)==[0 0];plot_type='2';disp('Individual ratings selected');end

% Specify roughness data location, min & max allowed shift, max number of iterations:
t35_95 = 2.03011;t1570_95=1.96148;t1570_90=1.64582;t1570_50 = .67465;
file_in='h:\RoughnessFinalAnalysis_08_15_2013.xls';sheet_in='Averages';data_range='c5:m84';
sheet_in2='CombinedRatings';data_range2a='a1:e1572';data_range2b='h1:l1572';
s_min=[0 0 0 0]';s_max=[2 5 35 15]';eps_stop=.000005;
x_max=[1.5 3.5 25 10;2.2 3.5 25 10];% Max parameter value to plot (row1:taxiway,row2:runway)

% Read data points
xls_data=xlsread(file_in,sheet_in,data_range);col=[2 3 4 6 8]';max_iter=100;
xls_data2a=xlsread(file_in,sheet_in2,data_range2a);
xls_data2b=xlsread(file_in,sheet_in2,data_range2b);
appr=['Taxiway ';'Runway  '];ISO=['Weighted RMS (m/s^2)       ';'Weighted MTVV (m/s^2)      ';'Weighted VDV (m/s^1^.^7^5)
';'DKup (m/s^2)              '];
unit = ones(37,1);unit2=ones(1572,1);%Column vector of ones

% Loop over taxiways and runways(kk = row 1 to begin taxiway data or row 38 to begin runway data)
for kk=1:37:38%kk=row where taxiway or runway data begins (1 or 38)
   b=xls_data(kk:kk+36,11);% b = 37 pilot ratings
```

```
crest = xls_data(kk:kk+36,9);
if kk==1
   b2=xls_data2a(1:1572,1);
else
   b2=xls_data2b(1:1572,1);
end
varb=var(b);
varb2=var(b2);
kk1=(kk-1)/37+1;%kk1=1 for taxiways, 2 for runways
% Loop over ISO parameters p=1:4
for p=1:4%p=1:Wt. RMS, p=2:Wt. MTVV, p=3: Wt. VDV, p=4: DKup
   fprintf(1,['Fit y = a*ln(x+s) + b for %8s With y=Pilot Rating Vs. x= %13s\n'],appr(kk1,1:8),ISO(p,1:27))
   fprintf(1,'-------------------------------------------------------------------------------\n')
   xx=[0:.1:x_max(kk1,p)];unit1=ones(size(xx));
   xi = xls_data(kk:kk+36,col(p));
   xc=xi(find(crest==1));

   % Find sqrt(r-squared) for min, max, and middle values of shift s to start the iteration
   s0 = s_min(p)*unit;
   A = [log(xi+s0),unit];% A = [ln(pth ISO parameter + s0) , 1]
   x0 = linsolve(A,b);% x0 = least squares solution of Ax = b using shift s0 by the QR method
   r = (b - A*x0);% r = residual vector (difference between data and fit)
   rsq0 = r'*r;% sum of squares of residuals

   s1 = s_max(p)*unit;
   A = [log(xi+s1),unit];% A = [ln(pth ISO parameter + s1) , 1]
   x1 = linsolve(A,b);% x1 = least squares solution of Ax = b using shift s1 by the QR method
   r = (b - A*x1);% r = r,esidual vector (difference between data and fit)
   rsq1 = r'*r;%sum of squares of residuals

   s = (s_min(p)+s_max(p))/2*unit;
   A = [log(xi+s),unit];% A = [ln(pth ISO parameter + s) , 1]
   x = linsolve(A,b);% x1 = least squares solution of Ax = b using shift s by the QR method
```

```
r = (b - A*x);% r = residual vector (difference between data and fit)
rsq = r'*r;% sum of squares of residuals
max_y=max([rsq0,rsq,rsq1]);
s_save = [s0(1) s1(1) s(1)];rsq_save = [rsq0 rsq1 rsq];

% Iteration to find the shift s that minimizes r-squared
disp(' Iteration          s          a          b              r^2              R^2   Max Error');
disp(' ---------  ----------  ----------  ----------  ----------------  ----------------   ---------');
for k=1:max_iter
   if abs(s1(1)-s0(1))>=eps_stop;
      if rsq1<=rsq0;if rsq<=rsq1;s0=s;rsq0=rsq;x0=x;else s0=s;rsq0=rsq;x0=x;s=s1;rsq=rsq1;x=x1;s1=2*s1-
s0;A=[log(xi+s1),unit];x1=linsolve(A,b);r = b-A*x1;rsq1=r'*r;end
      elseif rsq<=rsq0;s1=s;rsq1=rsq;x1=x;
      else s1=s;rsq1=rsq;x1=x;s=s0;rsq=rsq0;x=x0;s0=2*s0-s1;A=[log(xi+s0),unit];x0=linsolve(A,b);r=b-A*x0;rsq0=r'*r;
      end
      s = (s0+s1)/2;
      A = [log(xi+s),unit];
      x = linsolve(A,b);
      r = (b - A*x);
      rsq = r'*r;
      R2=1-rsq/varb/36;
      s_save = [s_save s(1)];rsq_save=[rsq_save rsq];
      fprintf(1,'%10i  %10f  %10f  %10f  %16.12f  %16.12f %10f\n',k,s(1),x(1),x(2),rsq,R2,s1(1)-s0(1));
   else
      break
   end
end

% Show the best fit
if k==max_iter;
   disp('*** Error - Failed to converge.\n\n');
else
   if rsq<=rsq1 && rsq<=rsq0
```

```
    R2=1-rsq/varb/36;
    fprintf(1,'Solution:   %10f  %10f  %10f  %16.12f  %16.12f %10f\n',s(1),x(1),x(2),rsq,R2,(s1(1)-s0(1))/2);
    a0=x(1);b0=x(2);shift=s(1);r2=rsq;
elseif rsq1<=rsq0
    R2=1-rsq1/varb/36;
    fprintf(1,'Solution:   %10f  %10f  %10f  %16.12f  %16.12f %10f\n',s1(1),x1(1),x1(2),rsq1,R2,s1(1)-s0(1));
    a0=x1(1);b0=x1(2);shift=s1(1);r2=rsq1;
else
    R2=1-rsq0/varb/36;
    fprintf(1,'Solution:   %10f  %10f  %10f  %16.12f  %16.12f %10f\n',s0(1),x0(1),x0(2),rsq0,R2,s1(1)-s0(1));
    a0=x0(1);b0=x0(2);shift=s0(1);r2=rsq0;
end

% Fill in the left side of the shift vs. r^2 plot
for ks=.1:.1:.7
    s = ks*shift*unit;
    A = [log(xi+s),unit];% A = [ln(pth ISO parameter + s) , 1]
    x = linsolve(A,b);% x1 = least squares solution of Ax = b using shift s by the QR method
    r = (b - A*x);% r = residual vector (difference between data and fit)
    rsq = r'*r;% sum of squares of residuals
    s_save = [s_save s(1)];rsq_save=[rsq_save rsq];
end

%Plot the best fit and show the shift yielding min. r^2
if plot_type(1)=='1' | plot_type(1)=='3'
  [s_save ind] = sort(s_save);% Sort shift iterates into ascending order
  rsq_save = rsq_save(ind);% Sort corresponding values of r^2
  R2=1-r2/varb/36;
  figure('Position',[11 11 900 600]);
  title({['Shifted Log Fit for ' appr(kk1,1:8) 'Avg. Rating Vs. ' ISO(p,1:13)],['y = ' num2str(a0) '*ln(x+' num2str(shift) ')+ '
num2str(b0)]},'FontSize',16,'Color','b');
  text(x_max(kk1,p)/8,9,['R^2=' num2str(R2)],'Color','b','FontSize',14)
  hold on
```

```
yy=a0*log(xx+shift*unit1) + b0*unit1;%y-values on fitted curve at xx x-values
y_bar=mean(b)*unit;%Average y-value of input data
xis=log(xi+shift*unit);%Log of shifted x-values of input data
x_bar=mean(xis)*unit;%Mean of log of shifted x-values of input data
steyx=sqrt(((b-y_bar)'*(b-y_bar)-((xis-x_bar)'*(b-y_bar))^2/((xis-x_bar)'*(xis-x_bar)))/35);%Standard error
temp=1/37*unit1 + (log(xx+shift*unit1)-x_bar(1)*unit1).^2/((xis-x_bar)'*(xis-x_bar));%
yy_conf95=t35_95*steyx*sqrt(temp);
yy_pred95=t35_95*steyx*sqrt((unit1+temp));
if plot_type(1)=='1'
   grid on;
   plot(xi,b,'bd',xx,yy,'m-',xx,yy-yy_conf95,'r--',xx,yy-yy_pred95,'r:',xx,yy+yy_conf95,'r--
',xx,yy+yy_pred95,'r:','LineWidth',3,'MarkerFaceColor','b');
   legend('37 Pilot Avg. Ratings','Fitted Curve','95% Confidence Interval','95% Prediction Interval');
   axis([0 x_max(kk1,p) 0 10]);
   xlabel(ISO(p,1:27));ylabel('Avg. Pilot Rating');
   figure('Position',[11 11 900 600]);grid on
   title(['Inverse of Shifted Log Fit: ' appr(kk1,1:8) ISO(p,1:13) ' Vs. Avg. Pilot Rating'],'FontSize',16,'Color','b');
   hold on
   plot(b,xi,'bd',yy,xx,'r--','MarkerFaceColor','b');
   axis([0 10 0 x_max(kk1,p)]);
   ylabel(ISO(p,1:27));xlabel('Avg. Pilot Rating');
   legend('37 Pilot Avg. Ratings','Fitted Curve');
   figure('Position',[11 11 900 600]);grid on
   title({['Residual (r^2) Vs. Shift (s)'],['For ' appr(kk1,1:8) ISO(p,1:13)]},'FontSize',16,'Color','b');
   hold on
   plot(s_save,rsq_save,'r.:',shift,r2,'r*');text(shift,r2-.25,[' (' num2str(shift) ',' num2str(r2) ')']);
   legend('r^2 of Least Squares Fit','Optimum Shift & Min. r^2');
   axis([min(s_save) max(s_save) min(rsq_save)-1 max(rsq_save)]);
   xlabel('s');ylabel('r^2');
else
   yc=b(find(crest==1));
   if p==1 %Extra plot vs. quadratic best fit predicted by Excel
```

```
                if kk==1;yq=2.9977*xx.*xx-11.089*xx+10.083*ones(size(xx));rr2=.9499;else;yq=1.7558*xx.*xx-
8.3822*xx+10.403*ones(size(xx));rr2=.9848;end%Quadratic fits from Excel
                plot(xi,b,'bd',xc,yc,'mo',xx,yy,'r-',xx,yq,'g-','LineWidth',2,'MarkerFaceColor','b');
                text(x_max(kk1,p)/8,9,['R^2=' num2str(R2)],'Color','r','FontSize',14)
                text(x_max(kk1,p)/8-.1,9.5,['R^2=' num2str(rr2)],'Color','g','FontSize',14)
                legend('37 Pilot Avg. Ratings','High crest factor case','Shifted Log. Fit','Quadratic Fit');
                axis([0 x_max(kk1,p) 0 10]);
                xlabel(ISO(p,1:27));ylabel('Avg. Pilot Rating');
                figure('Position',[11 11 900 600]);
                title({['Shifted Log Fit for ' appr(kk1,1:8) 'Avg. Rating Vs. ' ISO(p,1:13)],['y = ' num2str(a0) '*ln(x+' num2str(shift) ')+ '
num2str(b0)]},'FontSize',16,'Color','b');
                text(x_max(kk1,p)/8,9,['R^2=' num2str(R2)],'Color','b','FontSize',14)
            end
            % Plot high crest factor cases
            plot(xi,b,'bd',xc,yc,'mo',xx,yy,'r-','LineWidth',3,'MarkerFaceColor','b');
            legend('37 Pilot Avg. Ratings','High crest factor case','Fitted Curve');
            title({['Shifted Log Fit for ' appr(kk1,1:8) 'Avg. Rating Vs. ' ISO(p,1:13)],['y = ' num2str(a0) '*ln(x+' num2str(shift) ')+ '
num2str(b0)]},'FontSize',16,'Color','b');
            axis([0 x_max(kk1,p) 0 10]);
            xlabel(ISO(p,1:27));ylabel('Avg. Pilot Rating');
          end
        end

        % Fixed point iteration to find the shift that puts the best fit thru (0,10)
        fprintf(1,['\nFit y = a*ln(x+s) + b Through (0,10) for %8s With y=Pilot Rating Vs. x= %13s\n'],appr(kk1,1:8),ISO(p,1:27))
        disp(' Iteration          s          a          b              r^2              R^2');
        disp(' ---------  ----------  ----------  ----------  ----------------  ----------------');
        for kfp=1:1000
           s = exp((10-b0)/a0)*unit;
           if abs(s(1)-shift)<eps_stop;break;end
           A = [log(xi+s),unit];% A = [ln(pth ISO parameter + s) , 1]
           x = linsolve(A,b);% x1 = least squares solution of Ax = b using shift s by the QR method
           r = (b - A*x);% r = residual vector (difference between data and fit)
```

```
            rsq = r'*r;% sum of squares of residuals
            R2=1-rsq/varb/36;
            a0 = x(1);b0 = x(2);
            shift = s(1);
            %1if 10*floor(kfp/10)==kfp;fprintf(1,'%10i  %10f  %10f  %10f  %16.12f  %16.12f\n',kfp,shift,a0,b0,rsq,R2);end
        end
        fprintf(1,'%10i  %10f  %10f  %10f  %16.12f  %16.12f\n\n',kfp,shift,a0,b0,rsq,R2);
        disp(' ')
        if plot_type(1)=='1'
          figure('Position',[11 11 900 600]);grid on % Plot the fit through (0,10)
          title({['Shifted Log Fit Through (0,10) for ' appr(kk1,1:8) 'Avg. Rating Vs. ' ISO(p,1:13)],['y = ' num2str(a0) '*ln(x+'
num2str(shift) ')+ ' num2str(b0)]},'FontSize',16,'Color','b');
          text(x_max(kk1,p)/8,9,['R^2=' num2str(R2)],'Color','b','FontSize',14)
          hold on
          yy=a0*log(xx+shift*unit1) + b0*unit1;
          plot(xi,b,'bd',xx,yy,'r-','LineWidth',2,'MarkerFaceColor','b');
          axis([0 x_max(kk1,p) 0 10]);
          xlabel(ISO(p,1:27));ylabel('Avg. Pilot Rating');
          legend('37 Pilot Avg. Ratings','Fitted Curve');
        end

    end

    %Find the best fit to 1572 individual pilot responses
    fprintf(1,['Fit y = a*ln(x+s) + b to individual pilot responses for %8s With y=Pilot Rating Vs. x=
%13s\n'],appr(kk1,1:8),ISO(p,1:27))
    fprintf(1,'-----------------------------------------------------------------------------------------------------\n')
    if kk==1
        xii = xls_data2a(1:1572,p+1);
    else
        xii = xls_data2b(1:1572,p+1);
    end
```

```
% Find sqrt(r-squared) for min, max, and middle values of shift s to start the iteration
s0 = s_min(p)*unit2;
A = [log(xii+s0),unit2];% A = [ln(pth ISO parameter + s0) , 1]
x0 = linsolve(A,b2);% x0 = least squares solution of Ax = b using shift s0 by the QR method
r = (b2 - A*x0);% r = residual vector (difference between data and fit)
rsq0 = r'*r;% sum of squares of residuals

s1 = s_max(p)*unit2;
A = [log(xii+s1),unit2];% A = [ln(pth ISO parameter + s1) , 1]
x1 = linsolve(A,b2);% x1 = least squares solution of Ax = b using shift s1 by the QR method
r = (b2 - A*x1);% r = r,esidual vector (difference between data and fit)
rsq1 = r'*r;%sum of squares of residuals

s = (s_min(p)+s_max(p))/2*unit2;
A = [log(xii+s),unit2];% A = [ln(pth ISO parameter + s) , 1]
x = linsolve(A,b2);% x1 = least squares solution of Ax = b using shift s by the QR method
r = (b2 - A*x);% r = residual vector (difference between data and fit)
rsq = r'*r;% sum of squares of residuals
max_y=max([rsq0,rsq,rsq1]);
s_save = [s0(1) s1(1) s(1)];rsq_save = [rsq0 rsq1 rsq];

% Iteration to find the shift s that minimizes r-squared
disp(' Iteration          s          a          b              r^2              R^2   Max Error');
disp(' ---------  ----------  ----------  ----------  ----------------  ----------------   ---------');
for k=1:max_iter
   if abs(s1(1)-s0(1))>=eps_stop;
      if rsq1<=rsq0;if rsq<=rsq1;s0=s;rsq0=rsq;x0=x;else s0=s;rsq0=rsq;x0=x;s=s1;rsq=rsq1;x=x1;s1=2*s1-
s0;A=[log(xii+s1),unit2];x1=linsolve(A,b2);r = b2-A*x1;rsq1=r'*r;end
      elseif rsq<=rsq0;s1=s;rsq1=rsq;x1=x;
      else s1=s;rsq1=rsq;x1=x;s=s0;rsq=rsq0;x=x0;s0=2*s0-s1;A=[log(xii+s0),unit2];x0=linsolve(A,b2);r=b2-A*x0;rsq0=r'*r;
      end
      s = (s0+s1)/2;
      A = [log(xii+s),unit2];
```

```
        x = linsolve(A,b2);
        r = (b2 - A*x);
        rsq = r'*r;
        R2=1-rsq/varb2/1051;
        s_save = [s_save s(1)];rsq_save=[rsq_save rsq];
        fprintf(1,'%10i  %10f  %10f  %10f  %16.12f  %16.12f %10f\n',k,s(1),x(1),x(2),rsq,R2,s1(1)-s0(1));
    else
        break
    end
end

% Show the best fit
if k==max_iter;
    disp('*** Error - Failed to converge.\n\n');
else
    if rsq<=rsq1 && rsq<=rsq0
        R2=1-rsq/varb2/1051;
        fprintf(1,'Solution:   %10f  %10f  %10f  %16.12f  %16.12f %10f\n',s(1),x(1),x(2),rsq,R2,(s1(1)-s0(1))/2);
        a0=x(1);b0=x(2);shift=s(1);r2=rsq;
    elseif rsq1<=rsq0
        R2=1-rsq1/varb2/1051;
        fprintf(1,'Solution:   %10f  %10f  %10f  %16.12f  %16.12f %10f\n',s1(1),x1(1),x1(2),rsq1,R2,s1(1)-s0(1));
        a0=x1(1);b0=x1(2);shift=s1(1);r2=rsq1;
    else
        R2=1-rsq0/varb2/1051;
        fprintf(1,'Solution:   %10f  %10f  %10f  %16.12f  %16.12f %10f\n',s0(1),x0(1),x0(2),rsq0,R2,s1(1)-s0(1));
        a0=x0(1);b0=x0(2);shift=s0(1);r2=rsq0;
    end

    % Fill in the left side of the shift vs. r^2 plot
    for ks=.1:.1:.7
        s = ks*shift*unit2;
        A = [log(xii+s),unit2];% A = [ln(pth ISO parameter + s) , 1]
```

```
    x = linsolve(A,b2);% x1 = least squares solution of Ax = b using shift s by the QR method
    r = (b2 - A*x);% r = residual vector (difference between data and fit)
    rsq = r'*r;% sum of squares of residuals
    s_save = [s_save s(1)];rsq_save=[rsq_save rsq];
  end

  %Plot the best fit and show the shift yielding min. r^2
  if plot_type(1)=='2'
    [s_save ind] = sort(s_save);% Sort shift iterates into ascending order
    rsq_save = rsq_save(ind);% Sort corresponding values of r^2
    R2=1-r2/varb2/1051;
    % 95% Plot shifted log fit with confidence and prediction intervals
    figure('Position',[11 11 900 600]);grid on
    title({['Shifted Log Fit for ' appr(kk1,1:8) 'Individual Rating Vs. ' ISO(p,1:13)],['y = ' num2str(a0) '*ln(x+' num2str(shift) ')+ '
num2str(b0)]},'FontSize',16,'Color','b');
    text(x_max(kk1,p)/8,9,['R^2=' num2str(R2)],'Color','b','FontSize',14)
    hold on
    yy=a0*log(xx+shift*unit1) + b0*unit1;%y-values on fitted curve at xx x-values
    y_bar=mean(b2)*unit2;%Average y-value of input data
    xis=log(xii+shift*unit2);%Log of shifted x-values of input data
    x_bar=mean(xis)*unit2;%Mean of log of shifted x-values of input data
    steyx=sqrt(((b2-y_bar)'*(b2-y_bar)-((xis-x_bar)'*(b2-y_bar))^2/((xis-x_bar)'*(xis-x_bar)))/1570);%Standard error
    temp=1/1572*unit1 + (log(xx+shift*unit1)-x_bar(1)*unit1).^2/((xis-x_bar)'*(xis-x_bar));%
    yy_conf95=t1570_95*steyx*sqrt(temp);
    yy_pred95=t1570_95*steyx*sqrt((unit1+temp));
    plot(xii,b2,'b.',xx,yy,'m-',xx,yy-yy_conf95,'r--',xx,yy-yy_pred95,'r:',xx,yy+yy_conf95,'r--
',xx,yy+yy_pred95,'r:','LineWidth',3,'MarkerFaceColor','b');
    axis([0 x_max(kk1,p) 0 10]);
    legend('1572 Pilot Ratings','Fitted Curve','95% Confidence Interval','95% Prediction Interval');
    xlabel(ISO(p,1:27));ylabel('Avg. Pilot Rating');
    % Plot shifted log fit with 95%,90%,50% prediction intervals
    figure('Position',[11 11 900 600]);grid on
```

```
            title({['Shifted Log Fit & Prediction Intervals for ' appr(kk1,1:8) 'Individual Rating Vs. ' ISO(p,1:13)],['y = ' num2str(a0)
'*ln(x+' num2str(shift) ')+ ' num2str(b0)]},'FontSize',16,'Color','b');
            text(x_max(kk1,p)/3,9,['R^2=' num2str(R2)],'Color','k','FontSize',14)
            hold on
            yy_pred90=t1570_90*steyx*sqrt((unit1+temp));
            yy_pred50=t1570_50*steyx*sqrt((unit1+temp));
            plot(xii,b2,'b.',xx,yy,'k-','LineWidth',2)
            plot(xx,yy-yy_pred95,':','Color',[1 0 0],'LineWidth',3,'MarkerFaceColor','b')
            plot(xx,yy-yy_pred90,'-.','Color',[.95 .05 0],'LineWidth',3,'MarkerFaceColor','b')
            plot(xx,yy-yy_pred50,'--','Color',[.6 .25 .05],'LineWidth',3,'MarkerFaceColor','b')
            plot(xx,yy+yy_pred95,':','Color',[1 0 0],'LineWidth',3,'MarkerFaceColor','b')
            plot(xx,yy+yy_pred90,'-.','Color',[.95 .05 0],'LineWidth',3,'MarkerFaceColor','b')
            plot(xx,yy+yy_pred50,'--','Color',[.6 .25 .05],'LineWidth',3,'MarkerFaceColor','b')
            axis([0 x_max(kk1,p) 0 10]);
            legend('1572 Pilot Ratings','Fitted Curve','95% Prediction Interval','90% Prediction Interval','50% Prediction Interval');
            xlabel(ISO(p,1:27));ylabel('Avg. Pilot Rating');
            % Plot sqrt(r^2) as a function of shift & show minimizing shift
            figure('Position',[11 11 900 600]);grid on
            title({['Residual (r^2) Vs. Shift (s)'],['For ' appr(kk1,1:8) ISO(p,1:13)]},'FontSize',16,'Color','b');
            hold on
            plot(s_save,rsq_save,'r.:',shift,r2,'r*');text(shift,r2-.25,[' (' num2str(shift) ',' num2str(r2) ')']);
            legend('r^2 of Least Sq. Fit to Indiv. Resp.','Optimum Shift & Min. r^2');
            axis([min(s_save) max(s_save) min(rsq_save)-1 max(rsq_save)]);
            xlabel('s');ylabel('r^2');
          end
        end
    end
end
```